RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FIFTH SEMESTER EXAMINATION, DECEMBER 2016
THIRD YEAR [BATCH 2014-17]
PHYSICS [Honours]
Paper : V [Gr-C & D]

Date : 17/12/2016
Time : 11 am – 1 pm
Full Marks : 50

Group – C

(***Answer any three questions***) [3×10]

1. a) Consider the electron double slit experiment with a monochromatic photon source to determine which slit the electron went through. Can we actually see which slit the electron went through, maintaining the interference pattern? Does it help if we lower the intensity of the photons? [2]

   b) Classically, a free particle travels at a constant velocity. In the wave-packet description of the *quantum* free particle, does any part of the wave packet move with a constant velocity? Which velocity is this? [2]

   c) The wavelength of light emitted by a ruby LASER is 694.3 nm. Assuming the emission of a photon of this wavelength accompanies the transition of an electron from $n = 2$ level to the $n = 1$ level of an infinite square well, compute L for the well. [2]

   d) Suppose a wave-function of a three dimensional particle changes sign under a 2π rotation about the z axis; what values would the angular momentum quantum number l take in this case? (No derivation is required) [2]

   e) The wave function $\psi_0(x) = Ae^{-\frac{x^2}{2L^2}}$ represents the ground state energy of a harmonic oscillator.

   (i) Show that $\psi_1 = L\dfrac{d\psi_0(x)}{dx}$ is also a solution of Schrodinger equation.

   (ii) What is the energy of the new state? [2]

2. A quantum mechanical particle executes a one dimensional simple harmonic motion. Compute its *momentum space* wave function if the particle is in its first excited state. Also write down the full time dependent wave function for this state. Avoid using position space wave functions for the purpose of this problem. Use creation-annihilation operators instead. [6+4]

3. What is the general form of the spherical harmonic $Y_{\ell\ell}(\theta,\phi)$ as a function of its arguments? Obtain its normalization constant for $l = 2,3$. Determine the remaining spherical harmonics for $l = 2$ by applications of the lowering ladder operator $\hat{L}_-$, with appropriate normalizations. Can you think of a connection of this particular set of spherical harmonics with any multipole in electrostatics? [1+2+6+1]

4. Determine how the three dimensional quantum isotropic harmonic oscillator Hamiltonian $\hat{H} = \dfrac{\hat{p}^2}{2m} + \dfrac{1}{2}m\omega^2\hat{r}^2$ behaves under infinitesimal rotations in three dimensional space? In analogy with the 1 dimensional quantum harmonic oscillator discussed in class, can you write down the energy eigenvalue of this system? Do these energy eigenvalues depend on the angular momentum quantum numbers l, m? Why? [3+4+1+2]

5. a) If a planet in the solar system somehow loses its angular momentum, it crashes straight into the sun. What does the electron in a hydrogen atom do if it is in the $l = 0$ state? Why?

b) Hydrogen atom spectral lines have frequencies obeying the Ritz Combination Principle : $\omega_{mn} = \omega_{mp} + \omega_{pn}$, $m < p < n$. Show that this follows from the energy spectrum of the hydrogen atom derived in class.

c) Suppose the Coulomb potential of the hydrogen atom is changed by adding a term such that total potential is $V(r) = -\frac{Ze^2}{4\pi \in_0} \cdot \frac{1}{r} + \frac{K}{r^3}$ where K is a constant. On which quantum numbers its energy eigenvalues will depend? Qualitatively explain your answer. [1+2]

<u>Group – D</u>

(***Answer <u>any two</u> questions***) [2×10]

6. a) What are the basic conflicts of idea of 'spin' with classical theory? [2]
   b) What are good quantum numbers? How are they determined? [2+1]
   c) The ionisation energies of hydrogen (^{1}H) and lithium (^{3}Li) are 13.6 eV and 5.39 eV respectively. Determine, the effective nuclear charge experienced by the valence electrons of a ^{3}Li atom, in terms of the proton charge (e). [3]
   d) The emission wavelength for the transition $D_2 \to F_3$ is 312.2 nm. Determine the ratio of population of the final to the initial states at a temperature 5000K. [Given h = 6.626 × 10^{-34} J. sec., c = 3×10^8 m/sec and K = 1.380 × 10^{-23} J/K] [2]

7. a) How Bohr-Sommerfield model can explain the 'fine structure' of atomic spectra? [3]
   b) What is the main characteristic of the spectra of alkali atoms? How do this originate? Explain your answer briefly. [1+1+2]
   c) The electronic energy levels in a hydrogen atom are given by $E = -\frac{13.6}{n^2}$ eV. If a selective excitation to the n = 100 level is to be made using a LASER, determine the maximum allowed frequency line-width of the LASER. [3]

8. a) How does anomalous Zeeman effect differ from normal Zeeman effect. [1]
   b) The ground state of sodium atom (^{11}Na) is a $^2S_{1/2}$ state. Determine the difference in energy levels arising in presence of a weak external magnetic field (B), in terms of Bohr magneton μ_B. [2]
   c) Calculate the difference in energy levels (in eV) involved in transition of a Ruby LASER which emits 690 nm wavelength. [2]
   d) What do you mean by Optical resonator? [2]
   e) Define the P-Q-R branches of ro-vibrational spectra with corresponding selection rules. [3]

9. a) Derive an expression for the allowed rotational energy levels of an ideal rigid diatomic molecule. [4]
   b) What are the unique informations we obtain through the study of Raman spectra of a molecule? [2]
   c) Explain the basic working principle of solid state LASER with necessary diagram. [4]

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